Mechanics of materials

Chapter six

Bending

By

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Mechanics of materials

Bending

6.1. shear and moment diagrams

- Shear and moment diagrams are important to study the bending stresses.
- To construct shear moment diagram, follow these procedures

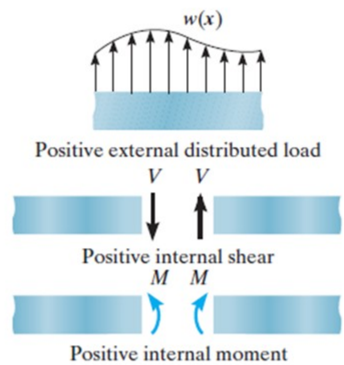
1. Find the supports reactions using F.B.D and static equilibrium equations
2. Derive both shear and moment functions with respect to x or y axis by analyzing sections through the body
3. Draw shear and moment diagrams from the equations you found in the previous step

Bending

Mechanics of materials

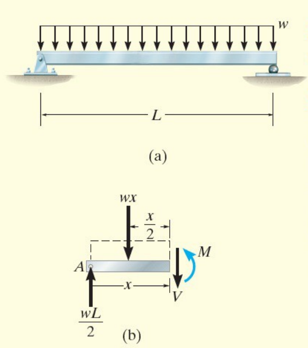
6.1. shear and moment diagrams

The sign conventions are opposite when the summing processes are carried out with opposite direction.



Beam sign convention

EXAMPLE 6.1



Draw the shear and moment diagrams for the beam shown in Fig. 6–4*a*.

SOLUTION

Support Reactions. The support reactions are shown in Fig. 6–4*c*.

Shear and Moment Functions. A free-body diagram of the left segment of the beam is shown in Fig. 6–4*b*. The distributed loading on this segment, wx, is represented by its resultant force only *after* the segment is isolated as a free-body diagram. This force acts through the centroid of the area comprising the distributed loading, a distance of $x/2$ from the right end. Applying the two equations of equilibrium yields

$$+\uparrow \Sigma F_y = 0; \qquad \frac{wL}{2} - wx - V = 0$$

$$V = w\left(\frac{L}{2} - x\right) \qquad (1)$$

$$\curvearrowleft + \Sigma M = 0; \qquad -\left(\frac{wL}{2}\right)x + (wx)\left(\frac{x}{2}\right) + M = 0$$

$$M = \frac{w}{2}(Lx - x^2) \qquad (2)$$

EXAMPLE 6.1 CONTINUED

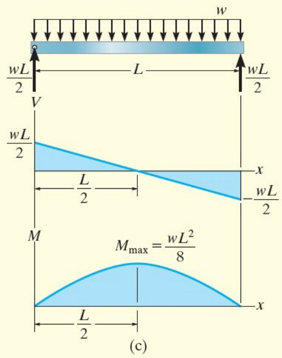


Fig. 6–4

Shear and Moment Diagrams. The shear and moment diagrams shown in Fig. 6–4*c* are obtained by plotting Eqs. 1 and 2. The point of *zero shear* can be found from Eq. 1:

$$V = w\left(\frac{L}{2} - x\right) = 0$$

$$x = \frac{L}{2}$$

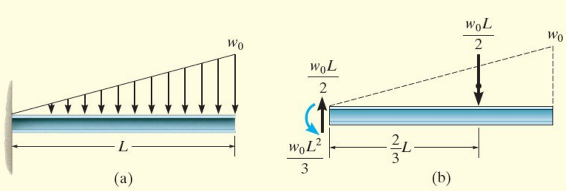
NOTE: From the moment diagram, this value of x represents the point on the beam where the *maximum moment* occurs, since by Eq. 6–2 (see Sec. 6.2) the *slope* $V = dM/dx = 0$. From Eq. 2, we have

$$M_{\max} = \frac{w}{2}\left[L\left(\frac{L}{2}\right) - \left(\frac{L}{2}\right)^2\right]$$

$$= \frac{wL^2}{8}$$

EXAMPLE 6.2

Draw the shear and moment diagrams for the beam shown in Fig. 6–5*a*.



SOLUTION

Support Reactions. The distributed load is replaced by its resultant force and the reactions have been determined as shown in Fig. 6–5*b*.

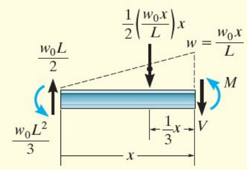
Shear and Moment Functions. A free-body diagram of a beam segment of length x is shown in Fig. 6–5*c*. Note that the intensity of the triangular load at the section is found by proportion, that is, $w/x = w_0/L$ or $w = w_0x/L$. With the load intensity known, the resultant of the distributed loading is determined from the area under the diagram. Thus,

EXAMPLE 6.2 CONTINUED

SOLUTION

Support Reactions. The distributed load is replaced by its resultant force and the reactions have been determined as shown in Fig. 6–5*b*.

Shear and Moment Functions. A free-body diagram of a beam segment of length x is shown in Fig. 6–5*c*. Note that the intensity of the triangular load at the section is found by proportion, that is, $w/x = w_0/L$ or $w = w_0x/L$. With the load intensity known, the resultant of the distributed loading is determined from the area under the diagram. Thus,



(c)

$$+\uparrow\Sigma F_y = 0; \qquad \frac{w_0L}{2} - \frac{1}{2}\left(\frac{w_0x}{L}\right)x - V = 0$$

$$V = \frac{w_0}{2L}(L^2 - x^2) \qquad (1)$$

EXAMPLE 6.2 CONTINUED

$$\circlearrowleft+\Sigma M = 0; \qquad \frac{w_0L^2}{3} - \frac{w_0L}{2}(x) + \frac{1}{2}\left(\frac{w_0x}{L}\right)x\left(\frac{1}{3}x\right) + M = 0$$

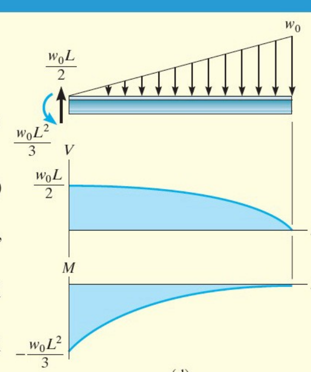
$$M = \frac{w_0}{6L}(-2L^3 + 3L^2x - x^3) \qquad (2)$$

These results can be checked by applying Eqs. 6–1 and 6–2 of Sec. 6.2, that is,

$$w = \frac{dV}{dx} = \frac{w_0}{2L}(0 - 2x) = -\frac{w_0x}{L} \qquad \text{OK}$$

$$V = \frac{dM}{dx} = \frac{w_0}{6L}(0 + 3L^2 - 3x^2) = \frac{w_0}{2L}(L^2 - x^2) \qquad \text{OK}$$

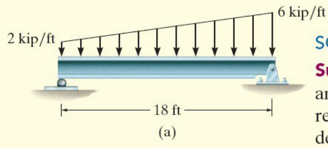
Shear and Moment Diagrams. The graphs of Eqs. 1 and 2 are shown in Fig. 6–5*d*.



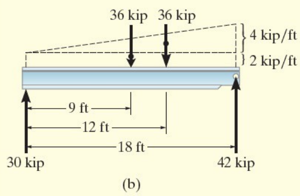
(d)

Fig. 6–5

EXAMPLE 6.3



Draw the shear and moment diagrams for the beam shown in Fig. 6–6*a*.



SOLUTION

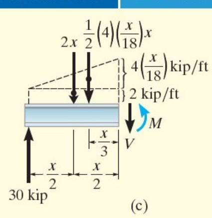
Support Reactions. The distributed load is divided into triangular and rectangular component loadings and these loadings are then replaced by their resultant forces. The reactions have been determined as shown on the beam's free-body diagram, Fig. 6–6*b*.

Shear and Moment Functions. A free-body diagram of the left segment is shown in Fig. 6–6*c*. As above, the trapezoidal loading is replaced by rectangular and triangular distributions. Note that the intensity of the triangular load at the section is found by proportion. The resultant force and the location of each distributed loading are also shown. Applying the equilibrium equations, we have

$$+\uparrow \Sigma F_y = 0;\quad 30\text{ kip} - (2\text{ kip/ft})x - \frac{1}{2}(4\text{ kip/ft})\left(\frac{x}{18\text{ ft}}\right)x - V = 0$$

$$V = \left(30 - 2x - \frac{x^2}{9}\right)\text{kip} \qquad (1)$$

EXAMPLE 6.3 CONTINUED



$$\curvearrowleft + \Sigma M = 0;$$

$$-30\text{ kip}(x) + (2\text{ kip/ft})x\left(\frac{x}{2}\right) + \frac{1}{2}(4\text{ kip/ft})\left(\frac{x}{18\text{ ft}}\right)x\left(\frac{x}{3}\right) + M = 0$$

$$M = \left(30x - x^2 - \frac{x^3}{27}\right)\text{kip}\cdot\text{ft} \qquad (2)$$

Equation 2 may be checked by noting that $dM/dx = V$, that is, Eq. 1. Also, $w = dV/dx = -2 - \frac{2}{9}x$. This equation checks, since when $x = 0$, $w = -2$ kip/ft, and when $x = 18$ ft, $w = -6$ kip/ft, Fig. 6–6*a*.

EXAMPLE 6.3 CONTINUED

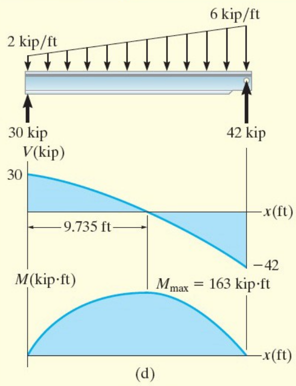


Fig. 6–6

Shear and Moment Diagrams. Equations 1 and 2 are plotted in Fig. 6–6*d*. Since the point of maximum moment occurs when $dM/dx = V = 0$ (Eq. 6–2), then, from Eq. 1,

$$V = 0 = 30 - 2x - \frac{x^2}{9}$$

Choosing the positive root,

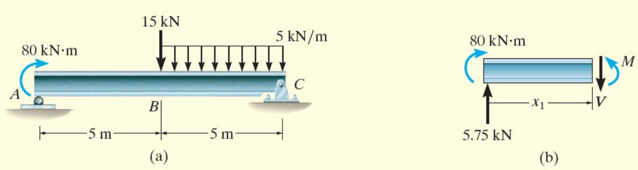
$$x = 9.735 \text{ ft}$$

Thus, from Eq. 2,

$$M_{max} = 30(9.735) - (9.735)^2 - \frac{(9.735)^3}{27}$$
$$= 163 \text{ kip} \cdot \text{ft}$$

EXAMPLE 6.4

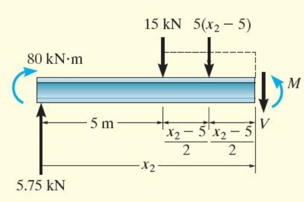
Draw the shear and moment diagrams for the beam shown in Fig. 6–7*a*.



SOLUTION

Support Reactions. The reactions at the supports have been determined and are shown on the free-body diagram of the beam, Fig. 6–7*d*.

Shear and Moment Functions. Since there is a discontinuity of distributed load and also a concentrated load at the beam's center, two regions of x must be considered in order to describe the shear and moment functions for the entire beam.



EXAMPLE 6.4 CONTINUED

$0 \le x_1 < 5$ m, Fig. 6–7*b*:

$+\uparrow \Sigma F_y = 0; \quad 5.75\text{ kN} - V = 0$

$$V = 5.75\text{ kN} \quad (1)$$

$\circlearrowleft + \Sigma M = 0; \quad -80\text{ kN}\cdot\text{m} - 5.75\text{ kN}\, x_1 + M = 0$

$$M = (5.75x_1 + 80)\text{ kN}\cdot\text{m} \quad (2)$$

$5\text{ m} < x_2 \le 10$ m, Fig. 6–7*c*:

$+\uparrow \Sigma F_y = 0; \quad 5.75\text{ kN} - 15\text{ kN} - 5\text{ kN/m}(x_2 - 5\text{ m}) - V = 0$

$$V = (15.75 - 5x_2)\text{ kN} \quad (3)$$

$\circlearrowleft + \Sigma M = 0; \quad -80\text{ kN}\cdot\text{m} - 5.75\text{ kN}\, x_2 + 15\text{ kN}(x_2 - 5\text{ m})$

$$+ 5\text{ kN/m}(x_2 - 5\text{ m})\left(\frac{x_2 - 5\text{ m}}{2}\right) + M = 0$$

$$M = (-2.5x_2^2 + 15.75x_2 + 92.5)\text{ kN}\cdot\text{m} \quad (4)$$

These results can be checked in part by noting that $w = dV/dx$ and $V = dM/dx$. Also, when $x_1 = 0$, Eqs. 1 and 2 give $V = 5.75$ kN and $M = 80$ kN·m; when $x_2 = 10$ m, Eqs. 3 and 4 give $V = -34.25$ kN and $M = 0$. These values check with the support reactions shown on the free-body diagram, Fig. 6–7*d*.

Shear and Moment Diagrams. Equations 1 through 4 are plotted in Fig. 6–7*d*.

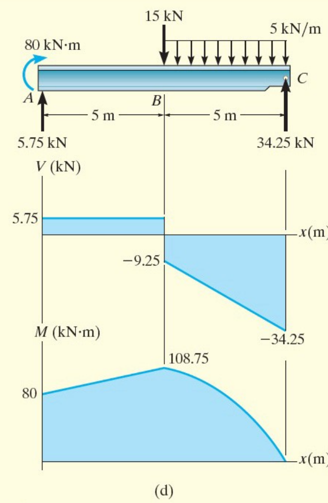


Fig. 6–7

M e c h a n i c s o f m a t e r i a l s

Bending

6.2. graphical method

Regions of distributed load:

Change in shear = area under distributed loading

$$\Delta V = -\int w(x)dx$$

Change in moment = area under shear diagram

$$\Delta M = \int V(x)dx$$

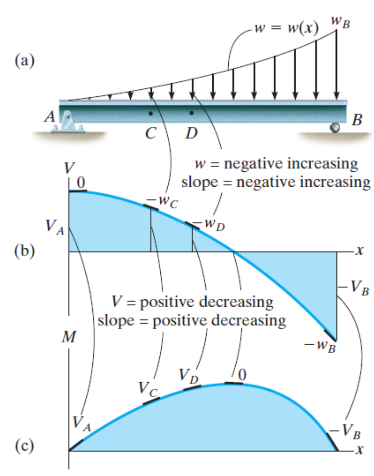
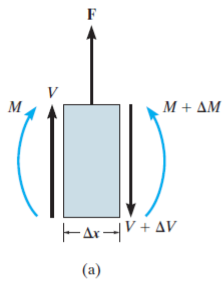


Fig. 6–9

Bending

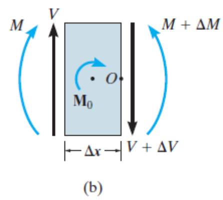
6.2. graphical method

Mechanics of materials



(a)

$$V - F - (V + \Delta V) = 0$$
$$\Delta V = -F$$

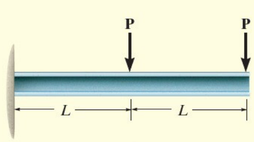


(b)

Fig. 6–10

$$M + \Delta M - M_0 - V\Delta x - M = 0$$
$$\Delta M = M_0$$

EXAMPLE 6.5



(a)

Draw the shear and moment diagrams for the beam shown in Fig. 6–11*a*.

SOLUTION

Support Reactions. The reaction at the fixed support is shown on the free-body diagram, Fig. 6–11*b*.

Shear Diagram. The shear at each end of the beam is plotted first, Fig. 6–11*c*. Since there is no distributed loading on the beam, the slope of the shear diagram is zero as indicated. Note how the force P at the center of the beam causes the shear diagram to jump downward an amount P, since this force acts downward.

Moment Diagram. The moments at the ends of the beam are plotted, Fig. 6–11*d*. Here the moment diagram consists of two sloping lines, one with a slope of $+2P$ and the other with a slope of $+P$.

The value of the moment in the center of the beam can be determined by the method of sections, or from the area under the shear diagram. If we choose the left half of the shear diagram,

$$M|_{x=L} = M|_{x=0} + \Delta M$$

EXAMPLE 6.5 CONTINUED

$$M|_{x=L} = -3PL + (2P)(L) = -PL$$

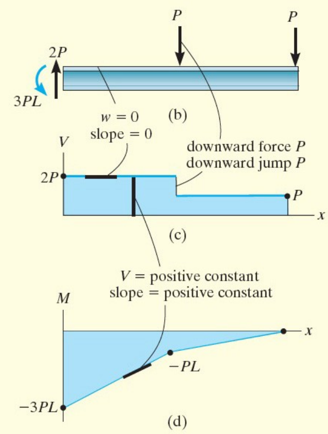
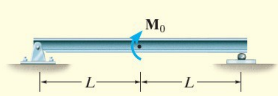


Fig. 6–11

EXAMPLE 6.6

Draw the shear and moment diagrams for the beam shown in Fig. 6–12*a*.



(a)

SOLUTION

Support Reactions. The reactions are shown on the free-body diagram in Fig. 6–12*b*.

Shear Diagram. The shear at each end is plotted first, Fig. 6–12*c*. Since there is no distributed load on the beam, the shear diagram has zero slope and is therefore a horizontal line.

Moment Diagram. The moment is zero at each end, Fig. 6–12*d*. The moment diagram has a constant negative slope of $-M_0/2L$ since this is the shear in the beam at each point. Note that the couple moment M_0 causes a jump in the moment diagram at the beam's center, but it does not affect the shear diagram at this point.

EXAMPLE 6.6 CONTINUED

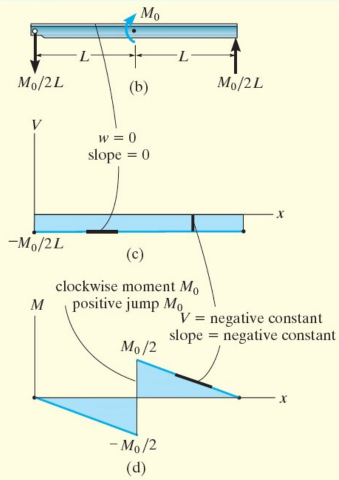


Fig. 6–12

EXAMPLE 6.7

Draw the shear and moment diagrams for each of the beams shown in Figs. 6–13*a* and 6–14*a*.

SOLUTION

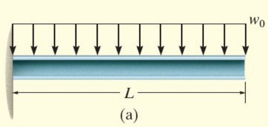
Support Reactions. The reactions at the fixed support are shown on each free-body diagram, Figs. 6–13*b* and Fig. 6–14*b*.

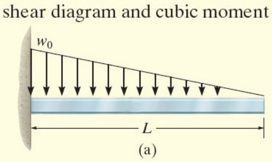
Shear Diagram. The shear at each end point is plotted first, Figs. 6–13*c* and 6–14*c*. The distributed loading on each beam indicates the slope of the shear diagram and thus produces the shapes shown.

Moment Diagram. The moment at each end point is plotted first, Figs. 6–13*d* and 6–14*d*. Various values of the shear at each point on the beam indicate the slope of the moment diagram at the point. Notice how this variation produces the curves shown.

EXAMPLE 6.7 CONTINUED

NOTE: Observe how the degree of the curves from w to V to M increases exponentially due to the integration of $dV = w\,dx$ and $dM = V dx$. For example, in Fig. 6–14, the linear distributed load produces a parabolic shear diagram and cubic moment diagram.





EXAMPLE 6.7 CONTINUED

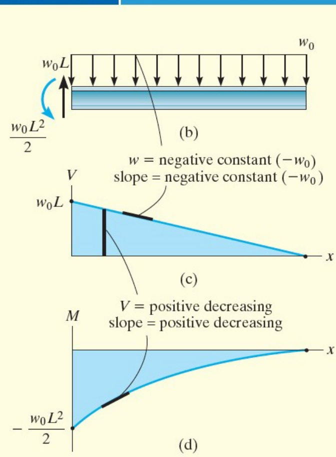


Fig. 6–13

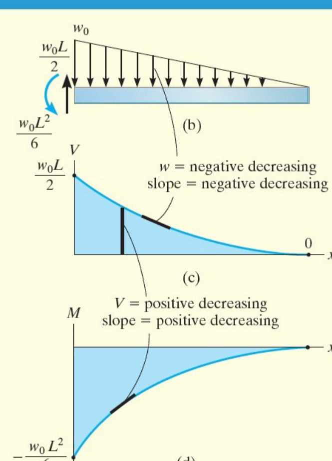
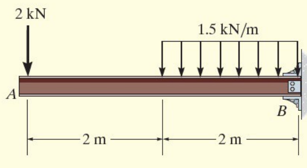


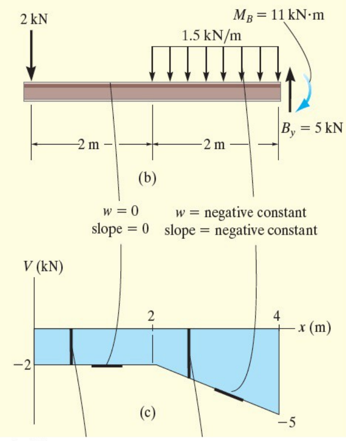
Fig. 6–14

EXAMPLE 6.8

Draw the shear and moment diagrams for the cantilever beam in Fig. 6–15*a*.



(a)



EXAMPLE 6.8 CONTINUED

SOLUTION

Support Reactions. The support reactions at the fixed support B are shown in Fig. 6–15*b*.

Shear Diagram. The shear at end A is -2 kN. This value is plotted at $x = 0$, Fig. 6–15*c*. Notice how the shear diagram is constructed by following the slopes defined by the loading w. The shear at $x = 4$ m is -5 kN, the reaction on the beam. This value can be verified by finding the area under the distributed loading, Eq. 6–3.

$$V\big|_{x=4\text{ m}} = V\big|_{x=2\text{ m}} + \Delta V = -2\text{ kN} - (1.5\text{ kN/m})(2\text{ m}) = -5\text{ kN}$$

Moment Diagram. The moment of zero at $x = 0$ is plotted in Fig. 6–15*d*. Notice how the moment diagram is constructed based on knowing its slope, which is equal to the shear at each point. The change of moment from $x = 0$ to $x = 2$ m is determined from the area under the shear diagram. Hence, the moment at $x = 2$ m is

$$M\big|_{x=2\text{ m}} = M\big|_{x=0} + \Delta M = 0 + [-2\text{ kN}(2\text{ m})] = -4\text{ kN}\cdot\text{m}$$

This same value can be determined from the method of sections, Fig. 6–15*e*.

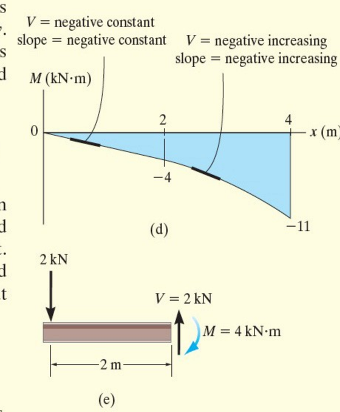
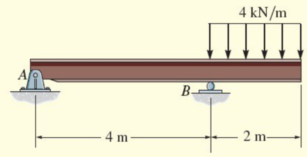


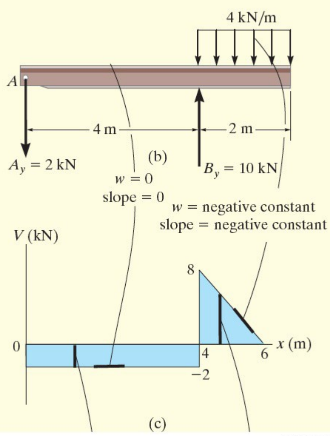
Fig. 6–15

EXAMPLE 6.9

Draw the shear and moment diagrams for the overhang beam in Fig. 6–16*a*.



(a)



(b)

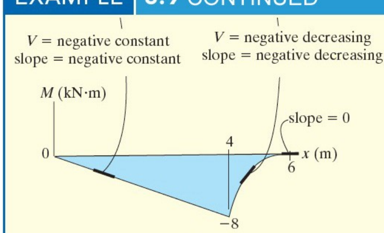
(c)

SOLUTION

Support Reactions. The support reactions are shown in Fig. 6–16*b*.

Shear Diagram. The shear of −2 kN at end A of the beam is plotted at $x = 0$, Fig. 6–16*c*. The slopes are determined from the loading and from this the shear diagram is constructed, as indicated in the figure. In particular, notice the positive jump of 10 kN at $x = 4$ m due to the force B_y, as indicated in the figure.

EXAMPLE 6.9 CONTINUED

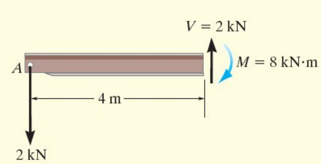


(d)

Moment Diagram. The moment of zero at $x = 0$ is plotted, Fig. 6–16*d*. Then following the behavior of the slope found from the shear diagram, the moment diagram is constructed. The moment at $x = 4$ m is found from the area under the shear diagram.

$$M|_{x=4\text{ m}} = M|_{x=0} + \Delta M = 0 + [-2\text{ kN}(4\text{ m})] = -8\text{ kN}\cdot\text{m}$$

We can also obtain this value by using the method of sections, as shown in Fig. 6–16*e*.

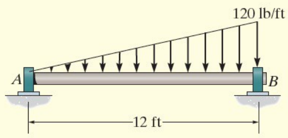


(e)

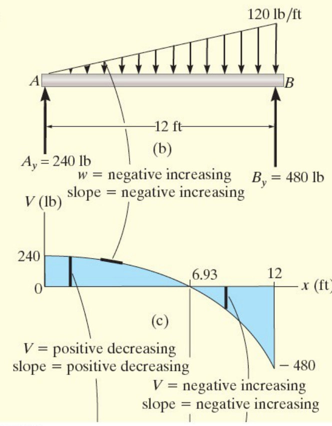
Fig. 6–16

EXAMPLE 6.10

The shaft in Fig. 6–17*a* is supported by a thrust bearing at *A* and a journal bearing at *B*. Draw the shear and moment diagrams.



(a)



EXAMPLE 6.10 CONTINUED

SOLUTION

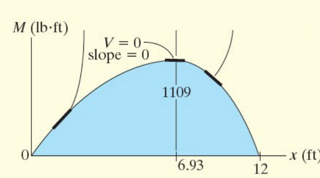
Support Reactions. The support reactions are shown in Fig. 6–17*b*.

Shear Diagram. As shown in Fig. 6–17*c*, the shear at $x = 0$ is +240. Following the slope defined by the loading, the shear diagram is constructed, where at *B* its value is −480 lb. Since the shear changes sign, the point where $V = 0$ must be located. To do this we will use the method of sections. The free-body diagram of the left segment of the shaft, sectioned at an arbitrary position *x*, is shown in Fig. 6–17*e*. Notice that the intensity of the distributed load at *x* is $w = 10x$, which has been found by proportional triangles, i.e., $120/12 = w/x$.

Thus, for $V = 0$,

$$+\uparrow \Sigma F_y = 0; \qquad 240 \text{ lb} - \tfrac{1}{2}(10x)x = 0$$

$$x = 6.93 \text{ ft}$$



(d)

EXAMPLE 6.10 CONTINUED

Moment Diagram. The moment diagram starts at 0 since there is no moment at A; then it is constructed based on the slope as determined from the shear diagram. The maximum moment occurs at $x = 6.93$ ft, where the shear is equal to zero, since $dM/dx = V = 0$, Fig. 6–17d,

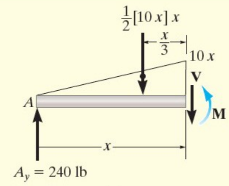
$$\circlearrowleft + \Sigma M = 0;$$

$$M_{\text{max}} + \tfrac{1}{2}[(10)(6.93)]\,6.93\left(\tfrac{1}{3}(6.93)\right) - 240(6.93) = 0$$

$$M_{\text{max}} = 1109 \text{ lb} \cdot \text{ft}$$

Finally, notice how integration, first of the loading w which is linear, produces a shear diagram which is parabolic, and then a moment diagram which is cubic.

NOTE: Having studied these examples, test yourself by covering over the shear and moment diagrams in Examples 6–1 through 6–4 and see if you can construct them using the concepts discussed here.



(e)

Fig. 6–17

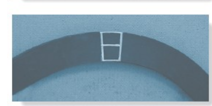
Bending

Mechanics of materials

6.3. bending deformation

Assumptions:

1. Plane section remains plane
2. Length of longitudinal axis remains unchanged
3. Plane section remains perpendicular to the longitudinal axis
4. In-plane distortion of section is negligible



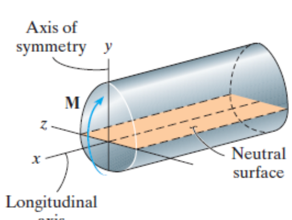


Fig. 6–18

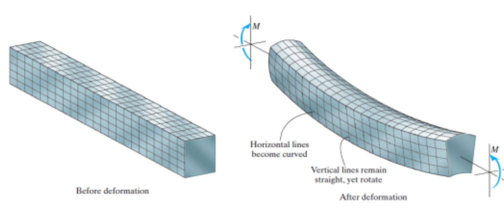


Fig. 6–19

Bending

Mechanics of materials

6.3. bending deformation

$$\sigma = -\frac{My}{I}$$

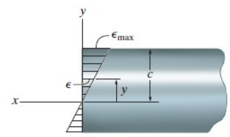
$$(M_R)_Z = \sum M_Z;$$

$$M = \int y\,dF = \int_A y(\sigma dA) = \int_A y\left(\frac{y}{c}\sigma_{max}\right)dA$$

$$M = \frac{\sigma_{max}}{c}\int_A y^2 dA$$

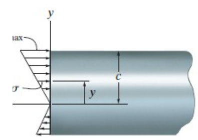
$$\sigma = -\frac{My}{I}$$

$$\sigma_{max} = \frac{Mc}{I}$$



Normal strain variation (profile view)

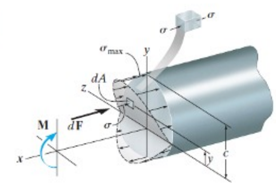
(a)



Bending stress variation (profile view)

(b)

Fig. 6–24



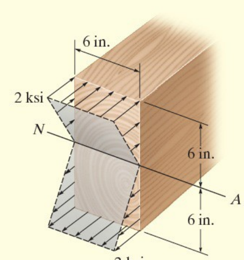
Bending stress variation

(c)

Fig. 6–24 (cont.)

EXAMPLE 6.11

A beam has a rectangular cross section and is subjected to the stress distribution shown in Fig. 6–25*a*. Determine the internal moment **M** at the section caused by the stress distribution (a) using the flexure formula, (b) by finding the resultant of the stress distribution using basic principles.



(a)

SOLUTION

Part (a). The flexure formula is $\sigma_{max} = Mc/I$. From Fig. 6–25*a*, $c = 6$ in. and $\sigma_{max} = 2$ ksi. The neutral axis is defined as line *NA*, because the stress is zero along this line. Since the cross section has a rectangular shape, the moment of inertia for the area about *NA* is determined from the formula for a rectangle given on the inside front cover; i.e.,

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(6\text{ in.})(12\text{ in.})^3 = 864\text{ in}^4$$

EXAMPLE 6.11 CONTINUED

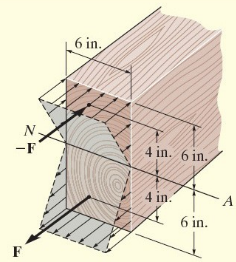
Therefore,

$$\sigma_{max} = \frac{Mc}{I}; \qquad 2\ \text{kip/in}^2 = \frac{M(6\ \text{in.})}{864\ \text{in}^4}$$

$$M = 288\ \text{kip}\cdot\text{in.} = 24\ \text{kip}\cdot\text{ft} \qquad \textit{Ans.}$$

Part (b). The resultant force for each of the two *triangular* stress distributions in Fig. 6–25*b* is graphically equivalent to the *volume* contained within each stress distribution. Thus, each volume is

$$F = \frac{1}{2}(6\ \text{in.})(2\ \text{kip/in}^2)(6\ \text{in.}) = 36\ \text{kip}$$



(b)

Fig. 6–25

EXAMPLE 6.11 CONTINUED

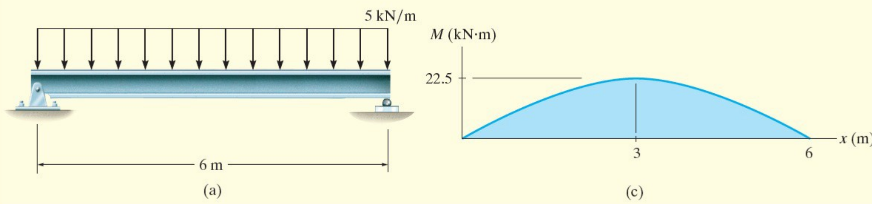
These forces, which form a couple, act in the same direction as the stresses within each distribution, Fig. 6–25*b*. Furthermore, they act through the *centroid* of each volume, i.e., $\frac{2}{3}(6\ \text{in.}) = 4$ in. from the neutral axis of the beam. Hence the distance between them is 8 in. as shown. The moment of the couple is therefore

$$M = 36\ \text{kip}(8\ \text{in.}) = 288\ \text{kip}\cdot\text{in.} = 24\ \text{kip}\cdot\text{ft} \qquad \textit{Ans.}$$

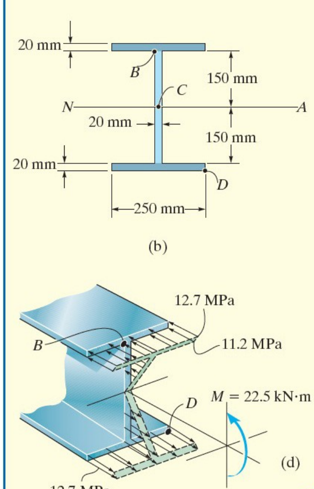
NOTE: This result can also be obtained by choosing a horizontal strip of area $dA = (6\ \text{in.})\ dy$ and using integration by applying Eq. 6–11.

EXAMPLE 6.12

The simply supported beam in Fig. 6–26*a* has the cross-sectional area shown in Fig. 6–26*b*. Determine the absolute maximum bending stress in the beam and draw the stress distribution over the cross section at this location.



EXAMPLE 6.12 CONTINUED



SOLUTION

Maximum Internal Moment. The maximum internal moment in the beam, $M = 22.5\ \text{kN} \cdot \text{m}$, occurs at the center.

Section Property. By reasons of symmetry, the neutral axis passes through the centroid *C* at the midheight of the beam, Fig. 6–26*b*. The area is subdivided into the three parts shown, and the moment of inertia of each part is calculated about the neutral axis using the parallel-axis theorem. (See Eq. A–5 of Appendix A.) Choosing to work in meters, we have

$$I = \Sigma(\bar{I} + Ad^2)$$

$$= 2\left[\frac{1}{12}(0.25\ \text{m})(0.020\ \text{m})^3 + (0.25\ \text{m})(0.020\ \text{m})(0.160\ \text{m})^2\right]$$

$$+ \left[\frac{1}{12}(0.020\ \text{m})(0.300\ \text{m})^3\right]$$

$$= 301.3(10^{-6})\ \text{m}^4$$

$$\sigma_{\max} = \frac{Mc}{I}; \qquad \sigma_{\max} = \frac{22.5(10^3)\ \text{N} \cdot \text{m}(0.170\ \text{m})}{301.3(10^{-6})\ \text{m}^4} = 12.7\ \text{MPa} \quad \textit{Ans.}$$

Fig. 6–26

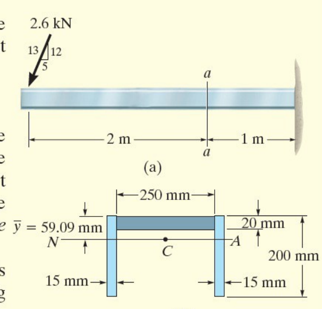
EXAMPLE 6.12 CONTINUED

A three-dimensional view of the stress distribution is shown in Fig. 6–26*d*. Notice how the stress at points *B* and *D* on the cross section develops a force that contributes a moment about the neutral axis that has the same direction as **M**. Specifically, at point *B*, $y_B = 150$ mm, and so

$$\sigma_B = -\frac{My_B}{I}; \qquad \sigma_B = -\frac{22.5(10^3)\ \text{N}\cdot\text{m}(0.150\ \text{m})}{301.3(10^{-6})\ \text{m}^4} = -11.2\ \text{MPa}$$

EXAMPLE 6.13

The beam shown in Fig. 6–27*a* has a cross-sectional area in the shape of a channel, Fig. 6–27*b*. Determine the maximum bending stress that occurs in the beam at section *a–a*.



SOLUTION

Internal Moment. Here the beam's support reactions do not have to be determined. Instead, by the method of sections, the segment to the left of section *a–a* can be used, Fig. 6–27*c*. In particular, note that the resultant internal axial force **N** passes through the centroid of the cross section. Also, realize that *the resultant internal moment must be calculated about the beam's neutral axis* at section *a–a*.

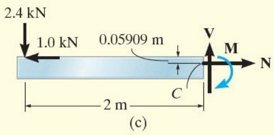
To find the location of the neutral axis, the cross-sectional area is subdivided into three composite parts as shown in Fig. 6–27*b*. Using Eq. A–2 of Appendix A, we have

$$\bar{y} = \frac{\Sigma\bar{y}A}{\Sigma A} = \frac{2[0.100\ \text{m}](0.200\ \text{m})(0.015\ \text{m}) + [0.010\ \text{m}](0.02\ \text{m})(0.250\ \text{m})}{2(0.200\ \text{m})(0.015\ \text{m}) + 0.020\ \text{m}(0.250\ \text{m})}$$

$$= 0.05909\ \text{m} = 59.09\ \text{mm}$$

This dimension is shown in Fig. 6–27*c*.

EXAMPLE 6.13 CONTINUED



(c)

Fig. 6–27

Applying the moment equation of equilibrium about the neutral axis, we have

$$\circlearrowleft + \Sigma M_{NA} = 0; \quad 2.4\text{ kN}(2\text{ m}) + 1.0\text{ kN}(0.05909\text{ m}) - M = 0$$

$$M = 4.859\text{ kN}\cdot\text{m}$$

Section Property. The moment of inertia about the neutral axis is determined using the parallel-axis theorem applied to each of the three composite parts of the cross-sectional area. Working in meters, we have

$$I = \left[\frac{1}{12}(0.250\text{ m})(0.020\text{ m})^3 + (0.250\text{ m})(0.020\text{ m})(0.05909\text{ m} - 0.010\text{ m})^2\right]$$

$$+ 2\left[\frac{1}{12}(0.015\text{ m})(0.200\text{ m})^3 + (0.015\text{ m})(0.200\text{ m})(0.100\text{ m} - 0.05909\text{ m})^2\right]$$

$$= 42.26(10^{-6})\text{ m}^4$$

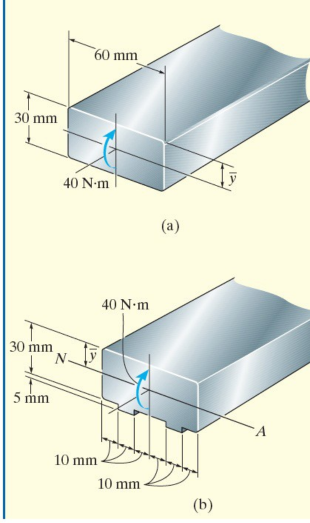
Maximum Bending Stress. The maximum bending stress occurs at points farthest away from the neutral axis. This is at the bottom of the beam, $c = 0.200\text{ m} - 0.05909\text{ m} = 0.1409\text{ m}$. Thus,

$$\sigma_{max} = \frac{Mc}{I} = \frac{4.859(10^3)\text{ N}\cdot\text{m}(0.1409\text{ m})}{42.26(10^{-6})\text{ m}^4} = 16.2\text{ MPa} \qquad \textit{Ans.}$$

Show that at the top of the beam the bending stress is $\sigma' = 6.79$ MPa.

NOTE: The normal force of $N = 1$ kN and shear force $V = 2.4$ kN will also contribute additional stress on the cross section. The superposition of all these effects will be discussed in Chapter 8.

EXAMPLE 6.14



The member having a rectangular cross section, Fig. 6–28*a*, is designed to resist a moment of 40 N · m. In order to increase its strength and rigidity, it is proposed that two small ribs be added at its bottom, Fig. 6–28*b*. Determine the maximum normal stress in the member for both cases.

SOLUTION

Without Ribs. Clearly the neutral axis is at the center of the cross section, Fig. 6–28*a*, so $\bar{y} = c = 15\text{ mm} = 0.015\text{ m}$. Thus,

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(0.060\text{ m})(0.030\text{ m})^3 = 0.135(10^{-6})\text{ m}^4$$

Therefore the maximum normal stress is

$$\sigma_{max} = \frac{Mc}{I} = \frac{(40\text{ N}\cdot\text{m})(0.015\text{ m})}{0.135(10^{-6})\text{ m}^4} = 4.44\text{ MPa} \qquad \textit{Ans.}$$

EXAMPLE 6.14 CONTINUED

With Ribs. From Fig. 6–28*b*, segmenting the area into the large main rectangle and the bottom two rectangles (ribs), the location $\bar{y}$ of the centroid and the neutral axis is determined as follows:

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A}$$

$$= \frac{[0.015\text{ m}](0.030\text{ m})(0.060\text{ m}) + 2[0.0325\text{ m}](0.005\text{ m})(0.010\text{ m})}{(0.03\text{ m})(0.060\text{ m}) + 2(0.005\text{ m})(0.010\text{ m})}$$

$$= 0.01592\text{ m}$$

This value does not represent c. Instead

$$c = 0.035\text{ m} - 0.01592\text{ m} = 0.01908\text{ m}$$

Using the parallel-axis theorem, the moment of inertia about the neutral axis is

$$I = \left[\frac{1}{12}(0.060\text{ m})(0.030\text{ m})^3 + (0.060\text{ m})(0.030\text{ m})(0.01592\text{ m} - 0.015\text{ m})^2\right]$$

$$+ 2\left[\frac{1}{12}(0.010\text{ m})(0.005\text{ m})^3 + (0.010\text{ m})(0.005\text{ m})(0.0325\text{ m} - 0.01592\text{ m})^2\right]$$

$$= 0.1642(10^{-6})\text{ m}^4$$

Therefore, the maximum normal stress is

$$\sigma_{max} = \frac{Mc}{I} = \frac{40\text{ N}\cdot\text{m}(0.01908\text{ m})}{0.1642(10^{-6})\text{ m}^4} = 4.65\text{ MPa} \qquad \textit{Ans.}$$

EXAMPLE 6.14 CONTINUED

This value does not represent c. Instead

$$c = 0.035\text{ m} - 0.01592\text{ m} = 0.01908\text{ m}$$

Using the parallel-axis theorem, the moment of inertia about the neutral axis is

$$I = \left[\frac{1}{12}(0.060\text{ m})(0.030\text{ m})^3 + (0.060\text{ m})(0.030\text{ m})(0.01592\text{ m} - 0.015\text{ m})^2\right]$$

$$+ 2\left[\frac{1}{12}(0.010\text{ m})(0.005\text{ m})^3 + (0.010\text{ m})(0.005\text{ m})(0.0325\text{ m} - 0.01592\text{ m})^2\right]$$

$$= 0.1642(10^{-6})\text{ m}^4$$

Therefore, the maximum normal stress is

$$\sigma_{max} = \frac{Mc}{I} = \frac{40\text{ N}\cdot\text{m}(0.01908\text{ m})}{0.1642(10^{-6})\text{ m}^4} = 4.65\text{ MPa} \qquad \textit{Ans.}$$

NOTE: This surprising result indicates that the addition of the ribs to the cross section will *increase* the normal stress rather than decrease it, and for this reason they should be omitted.